

# *On Circular Flows of Graphs*

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For an undirected graph  $G$ , the circular flow index of  $G$  is defined by

$$\phi_c(G) = \min_D \max_{\emptyset \neq X \subset V(G)} \frac{|\delta(X)|}{|\delta_D^+(X)|},$$

where the minimum is taken over all orientations of  $G$ . Galluccio and Goddyn in [Combinatorica, 22 (2002), 455-459] proved that if  $\kappa'(G) \geq 6$ , then  $\phi_c(G) < 4$ , using linear programming. We present a graph theory proof for the same result. Our result implies other family of graphs which may have edge-connectivity less than 6 can also have  $\phi_c(G) < 4$  ([Combinatorica, 27 (2007), 245-246]).