

Coloring Uniform Simple Hypergraphs with Few Edges

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A hypergraph H is b -simple if any two edges have at most b common vertices. Let $m(n, t, b)$ denote the minimum number of edges in a $(t+1)$ -chromatic n -uniform b -simple hypergraph. Erdos and Lovasz proved that $t^{2(n-2)}/[16n(n-1)^2] \leq m(n, t, 1) \leq 1600n^4t^{2(n+1)}$.

A result of Szabo improves the lower bound by a factor of n^{2-c} , where n is sufficiently large and c is a positive constant. Using similar techniques, we improve the lower bound by a further factor of n , where $c \rightarrow 0$ as $n \rightarrow \infty$. We also obtain bounds on $m(n, t, b)$ for $b \geq 2$.

Let $m'(n, t, g)$ denote the minimum number of edges in a $(t+1)$ -chromatic n -uniform hypergraph with girth g ; note that $m(n, t, 1) = m'(n, t, 3)$. We obtain lower bounds on $m'(n, t, g)$ for $g \geq 3$ and n sufficiently large; an upper bound was given by Erdos and Lovasz.