A Necessary and Sufficient Condition for a Tree to Be Graceful

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A tree T is graceful if it has a graceful labeling of its vertices. The graceful labeling of a tree T of order n is a one-to-one function from the set of vertices of T to the set $\{0, 1, 2, \ldots, n-1\}$ such that the induced edge labels are all distinct. In 1967, Rosa conjectured that all trees are graceful. This result has been proved only for few classes of trees.

Here we prove a necessary and sufficiency condition for a tree to be graceful, and use the technique to show that a certain class of spiders is graceful (a spider S is a tree with only one vertex v of $\deg(v) > 2$ such that the subgraph $S|_v$ is a disjoint union of paths).